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FREE VIBRATIONS OF A SLENDER BAR WITH NON-UNIFORM CHARACTERISTICS

by

Chi-Hung Mok
E. A. Murray, Jr.

December 1965

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Exterior Ballistics Laboratory

RDT&E Project No. 1P222901A201

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CHMok/EAMurray, Jr./blw
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ABSTRACT

The free vibrations of a slender bar with charactersitics slightly different from those of a uniform one is discussed. A simple approximate solution to this problem is presented. Results from experiments conducted with a bar of variable cross-section in free-free flexural vibrations indicate that the theory adequately predicts resonant frequencies and nodal locations.

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1. INTRODUCTION

The problem of free vibration of a slender, elastic bar has been treated by many authors in the past. Bars of variable cross-section and of non-homogeneous properties were investigated (See references on page 16). However, only in a small number of cases can a closed-form solution be obtained. In general, a tedious numerical procedure must be pursued to evaluate the natural frequencies and to determine the form of the principal modes. Here, it will be shown that, for slender bars with characteristics slightly deviated from those of a uniform bar, an approximate solution in closed-form is possible. The approximate solution is useful in evaluating the influence of non-uniformity on the natural frequencies and the principal modes.

2. LONGITUDINAL VIBRATION

For a slender bar whose cross-sectional area A, Young's modulus E, and mass density ρ, vary with the distance x along its axis, the equation of motion can be obtained as

$$\frac{\partial^2 u}{\partial x^2} + \left(\frac{A'}{A} + \frac{E'}{E}\right)\frac{\partial u}{\partial x} = \frac{\rho}{E}\frac{\partial^2 u}{\partial t^2} \qquad (1)$$

where u is the longitudinal displacement of a cross-section and the prime denotes a differentiation with respect to x. In the derivation of this equation, the state of stress in the bar is taken to be uniaxial in the direction of the bar axis and to be uniform over the whole cross-section. This approximation holds if the variation of the cross-sectional area is not great and if the wave length of the vibration is long compared to the lateral dimensions of the bar.

The vibratory solution of Equation (1) can be expressed in the form,

$$u = Xe^{i\omega t} \qquad (2)$$

in which X is a function of x only and i denotes $(-1)^{1/2}$. Thus, the frequency of the vibratory motion is equal to $\omega/2\pi$. The function X is the solution of the ordinary differential equation,

$$X'' + \left(\frac{A'}{A} + \frac{E'}{E}\right)X' + \frac{\rho}{E}\omega^2 X = 0 \tag{3}$$

subjected to a set of boundary conditions at the ends of the bar; i.e., at $x = 0$ and at $x = \ell$. The boundary conditions are

$$X = 0,$$

at a fixed end and (4)

$$X' = 0$$

at a free end.

A general solution for Equation (3) in terms of the functions A, E, ρ, is difficult to obtain. However, an approximate solution can be found if the area A, Young's modulus E, and the density ρ, vary only slightly with the position x, such that their second and higher derivatives and the products of their first derivatives may be neglected in comparison to the functions themselves. The approximate solution in its most general form is written as

$$X = G(D_1 \sin F + D_2 \cos F) \tag{5}$$

where G and the first derivative of F are functions weakly dependent on x (in the sense described above) and D_1 and D_2 are arbitrary constants to be determined from the boundary conditions at the ends. This form for the approximate solution is suggested by the solution for a uniform bar, i.e.,

$$X = B_1 \sin \frac{\omega x}{c} + B_2 \cos \frac{\omega x}{c} \tag{6}$$

where c is equal to $(E/\rho)^{1/2}$ and B_1, B_2 are arbitrary constants.

Using Equations (3) and (5) and neglecting all the higher order terms $\left((\frac{c}{\omega})^2 \frac{A'}{A}\frac{G'}{G}, (\frac{c}{\omega})^2 \frac{E'}{E}\frac{G'}{G} \text{ and } (\frac{c}{\omega})^2 \frac{G''}{G}\right)$ the following relations can be established,

$$G = (c/AE\omega)^{1/2} \tag{7}$$

and

$$F = \omega \int (1/c)dx \quad .$$

Accordingly, the approximate solution for X is

$$X = \left(\frac{c}{AE\omega}\right)^{1/2}\left(D_1 \sin \omega \int \frac{dx}{c} + D_2 \cos \omega \int \frac{dx}{c}\right) \quad . \tag{8}$$

It can be shown that the approximate solution holds closely if the dimensionless products $(\frac{c}{\omega}\frac{A'}{A})^2$, $(\frac{c}{\omega}\frac{E'}{E})^2$, $(\frac{c}{\omega}\frac{\rho'}{\rho})^2$; $(\frac{c}{\omega})^2\frac{A''}{A}$, $(\frac{c}{\omega})^2\frac{E''}{E}$, $(\frac{c}{\omega})^2\frac{\rho''}{\rho}$ are much less than unity.

The frequencies, $\omega/2\pi$, are obtained by satisfying the boundary conditions. It is interesting to note from Equations (8) and (4) that for all the boundary conditions considered here, the natural frequency is independent of the variation of cross-section away from the ends, but possibly dependent on the size and the taper of the bar at the ends. On the other hand, the variations of density and Young's modulus of the entire bar contribute to the value of the natural frequency through the integral term $\int \frac{dx}{c}$ in Equation (8).

The form of a normal mode of the vibration can be found by inserting into Equation (8) the values of the appropriate natural frequency. As one would have anticipated, the normal mode is no longer a simple harmonic in form as in the case of a uniform bar.

3. TORSIONAL VIBRATION

With a change of notation, the above discussion on longitudinal vibration can be applied to the study of the torsional vibration of a slender bar with a circular cross-section. For a bar whose cross-section area, shear modulus G, and density are functions of the distance x, the equation of motion for the torsional vibration is

$$\frac{\partial^2\theta}{\partial x^2} + \left(\frac{G'}{G} + \frac{J'}{J}\right)\frac{\partial\theta}{\partial x} = \frac{\rho}{G}\frac{\partial^2\theta}{\partial t^2} \tag{9}$$

where J is the polar moment of inertia of the cross-section about the bar axis and θ is the angle of rotation of a cross-section. This equation is identical to Equation (1) if G is replaced by E, J by A and θ by u. Consequently, the results obtained in the previous section hold also for the torsional vibration provided this change of notation is made.

4. FLEXURAL VIBRATION

On the basis of elementary beam theory, the equation of motion of a beam under free flexural vibration can be shown to be

$$\frac{\partial^4 W}{\partial x^4} + 2\left(\frac{E'}{E} + \frac{I'}{I}\right)\frac{\partial^3 W}{\partial x^3} + \left(\frac{E''}{E} + 2\,\frac{E'I'}{EI} + \frac{I''}{I}\right)\frac{\partial^2 W}{\partial x^2} = -\frac{\rho A}{EI}\frac{\partial^2 W}{\partial t^2} \qquad (10)$$

where W is the deflection of the neutral axis of the beam and I is the moment of inertia of the cross-section about the neutral axis. If Young's modulus E, the moment of inertia of the cross-section I, and the density ρ, are weakly dependent on the distance x, the same approach used previously in the study of longitudinal vibration can be applied to obtain an approximate solution for the flexural vibration. The approximation for the time independent part X, of the solution, $W = Xe^{i\omega t}$, is

$$X = (Q^3\omega^3E^2I^2)^{-1/4}(D_1 \cosh m + D_2 \sinh m + D_3 \cos m + D_4 \sin m) \qquad (11)$$

where

$$Q = \left(\frac{\rho A}{EI}\right)^{1/2}$$

$$m = \omega^{1/2}\int Q^{1/2}dx\ ,$$

and D_1, D_2, D_3 and D_4 are arbitrary constants. When A, I, E and ρ are constants, Equation (11) becomes the solution for a uniform bar, i.e.,

$$X = B_1 \cosh px + B_2 \sinh px + B_3 \cos px + B_4 \sin px \qquad (12)$$

where

$$p = \omega^{1/2}\left(\frac{\rho A}{EI}\right)^{1/4}$$

and B_1, B_2, B_3 and B_4 are arbitrary constants. The approximate solution will hold for the cases in which the dimensionless products $(A'/Ap)^2$, $(E'/Ep)^2$, $(\rho'/\rho p)^2$; A''/Ap^2, E''/Ep^2, ... ; ... , ... , ... ; A''''/Ap^4, ..., ... are much less than unity.

The frequencies and normal modes are determined from the boundary conditions at the two ends. The conditions are

$$X = 0,\ X'' = 0 \text{ for a simply supported end,}$$
$$X'' = 0,\ (EIX'')' = 0 \text{ for a free end,}$$
and
$$X = 0,\ X' = 0 \text{ for a clamped end.}$$

In contrast to longitudinal vibrations, it can be seen from Equation (11) that the flexural frequency of a bar is not free of the influence of the variation in cross-section away from the ends.

5. EXPERIMENTS

An experimental test of the analysis has been made. This was done by comparing the predicted and measured natural frequencies and nodal positions of a circular, stepped cross-section bar undergoing flexural vibrations with free-free end conditions. The specimen was aluminum and had a length of 31.42 inches. The diameter at the larger end was 1.387 inches and decreased by steps of 0.05 inches in the span to 0.798 inches at the smaller end; the stepped profile of the bar approximated that of a bar with continuously varying cross-sectional area determined by $1 + (1/2) \cos 0.1 x$, where x is the distance from the larger end along the center axis of the bar. For this cross-section, the condition given in Section 4 was satisfied; the largest value of the dimensionless products $(A'/Ap)^2$, ... etc., was 0.26 for the first mode and decreased to 0.05 for the third mode. The bar was suspended by two fine wires positioned at nodes and was excited electromagnetically at one end with a coil and magnet system. The coil, wound around the cylindrical surface of the bar, was connected to a sinusoidal current oscillator. Proximity of a permanent magnet to the coil provided the necessary mechanical excitation. Another wire winding and magnet system at the other end monitored the response of the bar. When the frequency of the sinusoidal excitation coincided with a natural frequency, the voltage output from the receiving winding showed a clear maximum. Nodes for each mode were detected with a phonograph cartridge.

Table I presents the measured and predicted values for the nodal positions and natural frequencies. Directing attention to the nodal positions, agreement between theory and experiment is seen to be acceptable and to be

best at the higher frequencies tested. This behavior probably stems from a lessening influence of the terms neglected in the analysis as the frequency increases.

The predicted and observed values of natural frequency are found to agree quite well. In general, however, the discrepancy between prediction and observation increases with frequency. This arises from the well-known inadequacies of a one-dimensional theory in describing the vibration of a slender bar. To assess this current analysis in the light of a comparable accepted theory, a test was conducted on a uniform aluminum bar with length equal to that of the stepped bar. The diameter of this second specimen was 0.99 inches which provides about the same length to diameter ratio found in the tapered bar if the latter's mean diameter is used.

Results from the test on the uniform bar are presented in Table II. The discrepancies between predicted and observed frequencies are seen to follow closely both in magnitude and general trend those found in the stepped bar. Hence, viewed in the light of a one-dimensional theory, the analysis developed here for bars of varying characteristics is seen to provide very good predictions of bar behavior.

6. CONCLUSION

The free vibration of a slender bar with characteristics slightly deviated from those of a uniform one has been discussed. Approximate but simple solutions are obtained for longitudinal, torsional and flexural vibrations. The solutions may be used for estimating the effects of small variations in the characteristics of a bar on the natural frequencies and on the modes of the vibrations. The results show that, in general, variations of characteristics of a bar produce a change in natural frequencies and a distortion in the form of normal modes from those of a uniform bar.

Among all the characteristics considered here, the variation in cross-sectional area has only localized effect on the vibrations of a bar except in the case of a flexural vibration. For longitudinal and torsional vibrations, the natural frequency is free from the influence of the size of

cross-sections away from the two ends. In addition, the amplitude of the normal mode at any section of the bar depends only on the size of that section.

The experimental results obtained for the free flexural vibration of a bar with a stepped profile indicate that this simple theory is adequate in predicting the natural frequencies and the positions of nodes. Neglecting higher order terms in the process of satisfying the differential equations and the boundary conditions does not seem to induce in the final results an error any larger than the order of magnitude of the abondoned terms. The accuracy of prediction improves at higher frequencies. Moreover, the results seem not to be affected by the presence of steps in the profile of the bar; hence, the local curvature in the non-uniformity of the bar may be overlooked in considering its free vibrations.

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CHI-HUNG MOK E. A. MURRAY, JR.

TABLE I

NATURAL FREQUENCIES AND NODES FOR SPECIMEN WITH VARIABLE CROSS-SECTION

Top Line Shows Predicted Results, Bottom Line, Measured Results

(Bar length = 31.42 inches)

Mode	Frequency (cycle/sec)	Discrepancy	Number of Nodes	Position of Nodes (inches from the large end)						
1	194	- 3%	2	7.89	25.27					
	200		2	6.69	23.81					
2	535	- 1%	3	4.68	17.06	27.83				
	538		3	4.38	16.13	27.44				
3	1050	+ 1%	4	3.35	12.37	21.47	28.86			
	1036		4	3.25	11.94	21.06	28.69			
4	1735	+ 3%	5	2.60	9.70	17.06	23.77	29.43		
	1687		5	2.31	9.44	16.75	23.5	29.31		
5	2592	+ 4%	6	2.13	7.97	14.14	19.87	25.21	29.79	
	2481		6	2.0	7.50	13.81	19.63	25.06	29.75	
6	3620	+ 6%	7	1.80	6.76	12.05	17.06	21.76	26.18	30.04
	3404		7	1.75	6.69	11.88	16.94	21.56	26.13	30.06

TABLE II

NATURAL FREQUENCIES AND NODES FOR SPECIMEN WITH UNIFORM CROSS-SECTION

Top Line Shows Predicted Results, Bottom Line, Measured Results

(Bar length = 31.42 inches)

Mode	Frequency (cycle/sec)	Discrepancy	Number of Nodes	Position of Nodes* (inches from one end)			
1	178	+ 2%	2	7.04	24.37		
	175		2	7.06	24.38		
2	491	+ 2%	3	4.15	15.71		
	481		3	4.13	15.69		
3	963	+ 3%	4	2.97	11.18		
	937		4	2.88	11.13		
4	1591	+ 3%	5	2.31	8.70	15.71	
	1538		5	2.25	8.69	15.69	
5	2377	+ 4%	6	1.89	7.11	12.85	
	2276		6	1.81	7.06	13.06	
6	3320	+ 6%	7	1.60	6.02	10.88	15.71
	3141		7	1.56	5.94	15.69	15.69

* *Only half of the nodes are shown because of symmetry about the center of the bar.*

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